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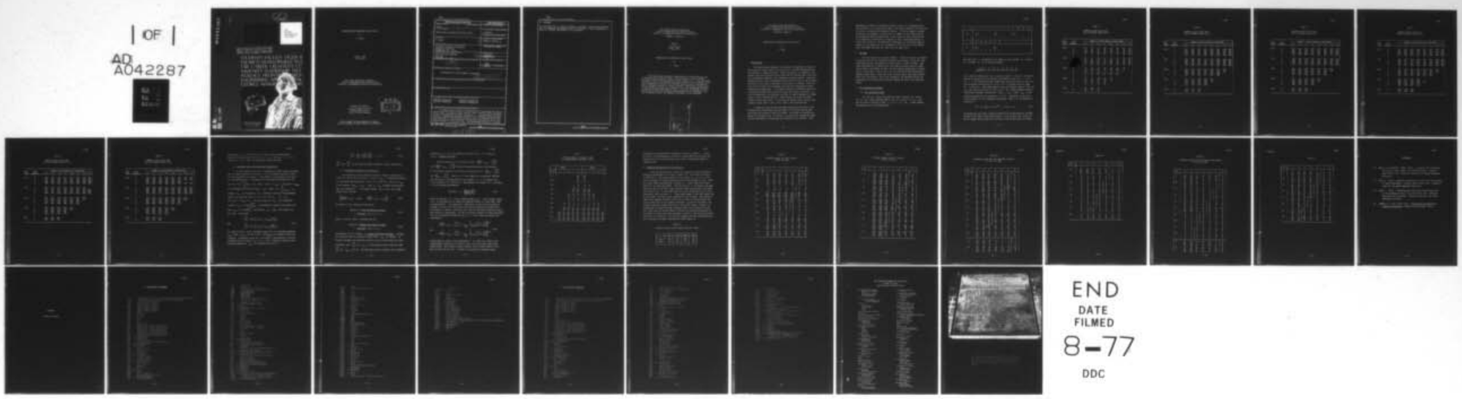

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FORECASTING THE RETENTION OF NAVY PILOTS

by

S. Zacks

Serial T-358
9 June 1977

The George Washington University
School of Engineering and Applied Science
Institute for Management Science and Engineering

Program in Logistics
Contract N00014-75-C-0729
Project NR 347 020
Office of Naval Research

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NONE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER T-358	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) FORECASTING THE RETENTION OF NAVY PILOTS		5. TYPE OF REPORT & PERIOD COVERED SCIENTIFIC
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) S. ZACKS		8. CONTRACT OR GRANT NUMBER(s) (15) N00014-75-C-0729
9. PERFORMING ORGANIZATION NAME AND ADDRESS THE GEORGE WASHINGTON UNIVERSITY PROGRAM IN LOGISTICS WASHINGTON, D.C. 20037 405 337		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS OFFICE OF NAVAL RESEARCH CODE 430D ARLINGTON, VA 22217		12. REPORT DATE (11) 9 JUNE 1977
		13. NUMBER OF PAGES 30
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (12) 34p.		15. SECURITY CLASS. (of this report) NONE
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED.

(14) Serial-T-358

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

MANPOWER FORECASTING	BINOMIAL DISTRIBUTION
ADAPTIVE ESTIMATION	RETENTION PROBABILITY

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The present paper develops a methodology for forecasting the size of specific officer groups (pilots) that will stay in service during specified future epochs (six months, twelve months, etc.). The statistical procedure is that of determining confidence intervals to the unknown retention probabilities, based on the observed retention rates, and applying these confidence intervals to obtain tolerance (prediction) intervals for the yet unobserved values of the group's size. The methodology developed is applied

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S/N 0102-014-6601

NONE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

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SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

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FORECASTING THE RETENTION OF NAVY PILOTS

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1. Introduction

The present paper addresses the problem of estimating the number of Navy pilots (or other career force officers of a certain profession) that are expected to remain in service. More specifically, we consider in the present study the retention of Navy pilots who are serving their first term in an operational squadron, after completing training schools. Generally, during the first term of operational service there is very little attrition. Attrition at this time is due mainly to accidents or sickness. After the first term of service the withdrawal of pilots from operational service becomes quite significant. It is highly important to develop a method for forecasting how many pilots from each cohort will remain in service. As will be shown later, the retention of pilots in service depends to a large extent on their original service. These sources are coded by certain original source codes (OSC). In our study we analyzed data from six such original sources (OSC 1, OSC 3, OSC 4, OSC 6, OSC 38 and OSC 80).

In addition, the data was classified according to the date of the minimum service requirement initials (MSRI) of the pilots and how many years they were before or after their MSRI. In Section 2 we provide six tables of the actual data. In Section 3 we present the statistical model and the methodology of forecasting the retention of the officers. This

methodology is based on determining tolerance limits for estimating prediction intervals of the anticipated number of personnel retained in the service. In Section 4 we present the numerical results. A Fortran computer program is given in the appendix. We would like to comment at this point that the forecasting procedure developed here is not restricted only to Navy pilots. It is expected to perform well if applied to other professional groups of career force officers. The theoretical development in the present paper is similar to that applied in the forecasting of the total size of the Marine Corps (see Haber and Zacks [1], Zacks [2], and Zacks [3]).

2. The Data

The data consist of six tables (Table 1 - Table 6) in which the number of pilots in service is tabulated according to the quarter of the year they attain MSRI and the time period (in years) before or after the MSRI. Each table corresponds to a different OSC, and spans the period from the beginning of 1971 to the middle of 1975. For the computer application we opened six data files (PIL 1, PIL 3, PIL 4, PIL 6, PI 38 and PI 80) consisting of eighteen lines (card). Each line (card) contains eight three-digit numbers. Lines 110 - 160 and 230 - 310 of the programs in the appendix provide the instructions for calling these files and reading the data in the specified manner.

3. The Forecasting Procedure

3.1 The Statistical Model

Let $X(i,j,t)$ denote the number of pilots from the ith source $(i = 1,\ldots,6)$, with specified MSRI at the jth quarter $(j = 1,\ldots,18)$ who are still in service at time t, $t = 1,\ldots,8$. The ... MSRI quarters are specified in the following table.

j	1	2	3	4	5	6	7	8	9	10	11	12
YR	1971				1972				1973			
Q	1	2	3	4	1	2	3	4	1	2	3	4
j	13	14	15	16	17	18						
YR	1974				1975							
Q	1	2	3	4	1	2						

The time index t corresponds to the number of years before (-) or after (+) the MSRI (o) in the following manner:

t	1	2	3	4	5	6	7	8
Years	-2.	-1.	-.5	0	0.5	1.	1.5	2.

For a specific combination (i,j), given the value of $X(i,j,t)$ we wish to predict the values that this variable will assume at time $t+v$, $v = 1,2,3,4,\ldots$. The statistical model assumes that, given the value of $X(i,j,t)$, the conditional distribution of $X(i,j,t+v)$ is binomial over the range $\{0,\ldots,X(i,j,t)\}$. More specifically, given the observed value of $X(i,j,t)$, n_{ijt} say, the future value of $X(i,j,t+v)$, is a random variable having a (conditional) binomial distribution, $B(n_{ijt}, \theta_{itv})$, where θ_{itv} is the retention probability for the period $(t, t+v)$. Generally, we say that a random variable X has a binomial distribution $B(N,\theta)$ if its probability function is

$$P[X = j] = \binom{N}{j} \theta^j (1-\theta)^{N-j}, \quad j = 0,1,\ldots,N . \tag{3.1}$$

We assume here that these retention probabilities do <u>not</u> depend on the MSRI quarter of the cohort under consideration. In other words, all cohorts of pilots coming from the same OSC at different time periods have, at similar

TABLE 1

NUMBER OF NAVY PILOTS FROM
OSC 1 IN SERVICE (FILE PIL 1)

MSRI Year	Date Quarter	Number of Years Before or After MSRI							
		-2.	-1.	-.5	0	0.5	1.	1.5	2.
1971	1	57	57	57	48	42	42	41	41
	2	25	25	24	20	19	19	18	16
	3	22	21	21	15	14	14	13	12
	4	97	97	96	85	78	72	68	58
1972	1	47	47	46	41	41	37	33	30
	2	22	22	22	21	18	17	15	14
	[illegible]	57	56	56	53	48	45	37	36
	[illegible]	125	124	122	111	102	95	80	78
1973	1	57	55	55	51	43	35	35	
	2	24	23	23	22	21	20	17	
	3	8	8	8	7	7	7		
	4	177	176	171	155	130	124		
1974	1	33	31	30	27	27			
	2	12	12	12	10	8			
	3	9	9	8	8				
	4	74	74	72	62				
1975	1	66	66	65					
	2	56	54	54					

TABLE 2

NUMBER OF NAVY PILOTS FROM
OSC 3 IN SERVICE (FILE PIL 3)

MSRI Year	Date Quarter	Number of Years Before or After MSRI							
		-2.	-1.	-.5	0	0.5	1.	1.5	2.
1971	1	121	118	071	047	046	044	039	038
	2	140	085	077	060	058	052	050	042
	3	171	140	137	084	073	066	058	053
	4	117	113	107	063	060	051	044	042
1972	1	192	189	182	120	110	089	084	077
	2	258	244	224	144	128	108	097	087
	3	243	234	186	122	105	091	076	070
	4	179	169	150	091	076	070	061	059
1973	1	181	146	130	083	066	057	049	
	2	296	243	207	137	121	108	105	
	3	162	125	113	077	065	064		
	4	227	172	145	111	092	088		
1974	1	235	173	160	109	094			
	2	341	242	240	184	177			
	3	171	132	127	090				
	4	171	152	134	116				
1975	1	179	172	167					
	2	174	173	170					

TABLE 3

NUMBER OF NAVY PILOTS FROM
OSC 4 IN SERVICE (FILE PIL 4)

MSRI Year	Date Quarter	Number of Years Before or After MSRI							
		-2.	-1.	-.5	0	0.5	1.	1.5	2.
1971	1	037	036	035	028	022	021	020	019
	2	035	035	032	025	024	023	023	020
	3	066	066	064	051	046	046	042	040
	4	095	089	088	071	065	061	057	050
1972	1	042	040	040	034	029	024	023	020
	2	038	037	037	034	032	031	028	026
	3	056	056	056	052	049	046	034	034
	4	078	074	074	065	058	051	041	036
1973	1	041	040	040	032	031	023	018	
	2	047	047	045	036	031	026	025	
	3	093	092	088	07[illegible]	057	052		
	4	037	037	035	032	024	021		
1974	1	012	012	012	011	010			
	2	024	024	024	018	015			
	3	064	059	057	043				
	4	060	055	052	046				
1975	1	032	031	028					
	2	034	032	032					

TABLE 4

NUMBER OF NAVY PILOTS FROM
OSC 6 IN SERVICE (FILE PIL 6)

MSRI Year	Date Quarter	Number of Years Before or After MSRI							
		-2.	-1.	-.5	0	0.5	1.	1.5	2.
1971	1	011	011	007	006	005	003	003	003
	2	021	019	018	013	012	011	010	009
	3	022	022	021	013	013	013	013	013
	4	016	016	015	011	010	009	009	009
1972	1	016	016	015	008	007	007	005	005
	2	020	017	017	012	011	008	008	007
	3	017	015	013	010	010	008	008	008
	4	007	007	006	004	003	003	003	003
1973	1	018	015	013	008	007	007	007	
	2	033	029	027	018	014	013	013	
	3	022	015	014	010	010	009		
	4	008	005	004	003	002	001		
1974	1	006	004	004	002				
	2	006	003	003	002	002			
	3	005	004	003	002				
	4	006	006	005	005				
1975	1	002	002	002					
	2	002	002	002					

TABLE 5

NUMBER OF NAVY PILOTS FROM
OSC 38 IN SERVICE (FILE PL 38)

MSRI Year	Date Quarter	Number of Years Before or After MSRI							
		-2.	-1.	-.5	0	0.5	1.	1.5	2.
1971	1	014	014	010	007	008	008	008	008
	2	023	016	016	011	011	010	010	009
	3	014	014	014	011	010	010	008	008
	4	006	006	005	003	003	002	002	002
1972	1	005	005	005	002	001	001	001	
	2	017	017	017	013	013	011	011	010
	3	013	012	011	008	007	007	006	006
	4	004	003	003	002	002	002	002	002
1973	1	020	018	016	012	011	010	010	
	2	017	016	013	007	007	007	007	
	3	013	011	010	009	008	008		
	4	012	010	009	006	006	006		
1974	1	013	010	010	008	007			
	2	021	018	018	013	013			
	3	008	007	006	006				
	4	007	006	004	003				
1975	1	002	001	001					
	2	001	001	001					

TABLE 6

NUMBER OF NAVY PILOTS FROM
OSC 80 IN SERVICE (FILE PL 80)

MSRI Year	Date Quarter	Number of Years Before or After MSRI							
		-2.	-1.	-.5	0	0.5	1.	1.5	2.
1971	1	002	002	001	001	001	001	001	001
	2	002	002	001	001	001	001	001	001
	3	005	003	003	002	002	001		
	4	011	009	008	006	006	006	005	004
1972	1	006	006	006	005	004	002	002	001
	2	005	005	004	003	003	003	003	003
	3	019	018	017	015	013	013	010	010
	4	040	036	031	026	023	019	016	013
1973	1	022	020	018	013	010	010	008	
	2	021	019	019	016	014	013	013	
	3	053	041	036	024	018	018		
	4	064	045	036	030	024	022		
1974	1	034	029	027	019	019			
	2	039	026	026	021	021			
	3	020	018	017	014				
	4	016	014	013	011				
1975	1	031	028	027					
	2	055	054	055					

time points, t, of their service, the same retention probabilities. Moreover, we assume that, for each $i = 1,\ldots,6$, and each $t = 1,\ldots,8$; $\{X(i,j,t) ; j = 1,\ldots,18\}$ are independent random variables.

3.2 Confidence Limits For Retention Probabilities

It is well known that the sum of independent binomial random variables, with the same probability of success p, has a binomial distribution with the same probability of success p and number of trials which is the sum of the corresponding ones. Accordingly, the conditional distribution of the sum $T(i, t+v) = \sum_{j=1}^{18} X(i,j, t+v)$ given $\{X(i,1,t) = n_{i1t},\ldots,X(i,18,t) = n_{i18t}\}$ is the binomial distribution $B(N_{it}, \theta_{itv})$, where $N_{it} = \sum_{j=1}^{18} n_{ijt}$. We estimate θ_{itv} by determining $(1-\alpha)$ confidence limits for this parameter based on the observed values of $T(i, t+v)$ and N_{it}. Let $\hat{\theta}_{itv} = (T(i, t+v) + .5)/N_{it} + 1)$. For large values of N_{it}, the transformed variable $Y_{itv} = 2 \arcsin \sqrt{\hat{\theta}_{itv}}$ is approximately normally distributed with mean $\eta_{itv} = 2 \arcsin \sqrt{\hat{\theta}_{itv}}$ and variance $D_{it} = 1/N_{it}$ (See Johnson and Kotz [4]). Accordingly

$$Y_{itv}^{(1)} = \max\left(0, Y_{itv} - Z_{1-\alpha/2}\Big/\sqrt{N_{it}}\right)$$

and

$$Y_{itv}^{(2)} = \min\left(\pi, Y_{itv} + Z_{1-\alpha/2}\Big/\sqrt{N_{it}}\right) \tag{3.2}$$

are lower and upper $(1-\alpha)$ confidence limits for the transformed parameter η_{itv}; where $Z_{1-\alpha/2}$ is the $(1-\alpha/2)$ fractile of the standard normal distribution. Furthermore, since the $2 \arcsin\sqrt{\theta}$ transformation is strictly increasing over the range of $0 \leq \theta \leq 1$, $(1-\alpha)$ confidence limits for the retention probability θ_{itv} are obtained from (3.2) by

$$\theta^{(k)}_{itv} = \left[\sin\left(Y^{(k)}_{itv}/2\right)\right]^2 , \quad k = 1,2 . \tag{3.3}$$

$\theta^{(1)}_{itv}$ and $\theta^{(2)}_{itv}$ are the lower and upper confidence limits, respectively.

3.3 Prediction Intervals For Forecasting

On the basis of these confidence limits we compute a prediction interval for $X(i,j, t+v)$ given that $X(i,j,t) = n_{ijt}$. As mentioned earlier, the conditional distribution of $X(i,j, t+v)$ given that $X(i,j,t) = n_{ijt}$ is the binomial $B(n_{ijt} , \theta_{itv})$. Thus, if θ_{itv} is known, we anticipate that with probability γ the random variable $X(i,j, t+v)$ will assume values in the interval

$$\left[BL\left(\frac{1-\gamma}{2} ; n_{ijt} , \theta_{itv}\right) , \quad BU\left(\frac{1+\gamma}{2} ; n_{ijt} , \theta_{itv}\right)\right] . \tag{3.4}$$

The limits of this interval are defined by

$$\begin{aligned} BU(p;N,\theta) &= \underline{\text{least non-negative integer}}, \\ & j , \underline{\text{such that}} \ P_\theta[X \le j] \ge p ; \end{aligned} \tag{3.5}$$

where X has the $B(N,\theta)$ distribution; and

$$\begin{aligned} BL(p;N,\theta) &= \underline{\text{maximal non-negative integer}}, \\ & j , \underline{\text{such that}} \ P_\theta[X \le j] \le p . \end{aligned} \tag{3.6}$$

The interval (3.4) is called a <u>γ - content prediction interval</u>. In almost all practical cases the retention probabilities θ_{itv} are unknown. We therefore estimate the prediction interval (3.4) by substituting the lower confidence limit $\theta^{(1)}_{itv}$ for θ_{itv} in (3.6) and the upper confidence limit $\theta^{(2)}_{itv}$ for θ_{itv} in (3.3). The resulting interval contains, with confidence

probability $1-\alpha$, the true prediction interval (3.4). It is called a $(1-\alpha,\gamma)$ tolerance interval.

The determination of the tolerance limits $BL\left(\frac{1-\gamma}{2}\,;\, n_{ijt}\,,\, \theta^{(1)}_{itv}\right)$ and $BU\left(\frac{1+\gamma}{2}\,;\, n_{ijt}\,,\, \theta^{(2)}_{itv}\right)$ follows an algorithm based on the evaluation of the binomial cumulative distribution functions (c.d.f.) $B\left(n_{ijt}\,,\, \theta^{(1)}_{itv}\right)$ and $B\left(n_{ijt}\,,\, \theta^{(2)}_{itv}\right)$. These c.d.f.'s are computed by a subroutine FUNCTION CB(I,N,P) given in program NAVYS of the appendix. If N is greater than 50 the binomial c.d.f. CB(I,N,P) is approximated by a normal c.d.f., according to the well-known approximation

$$CB(j,N,p) \approx \Phi\left(\frac{j+.5 - Np}{\sqrt{Np(1-p)}}\right) \;; \tag{3.7}$$

where the function $\Phi(\cdot)$ is the standard normal c.d.f.. This standard normal c.d.f. is determined by the subroutine FUNCTION CNDX(Y) (in lines 1820 - 2010 of program NAVYS). The algorithm of computation is constructed according to Formulae (3.5) and (3.6) (see subroutine function KBL(P,N,T) and KBU(P,N,T). The program is, however, relatively slow since the subroutine functions require many repetitive computations. We have, therefore, applied the normal approximation to the binomial distribution even if N is not large, and applied the approximation

$$BL\left(\frac{1-\gamma}{2}\,;\, n_{ijt}\,,\, \theta^{(1)}_{itv}\right) \approx Z_{\frac{1-\gamma}{2}} \left\{n_{ijt}\theta^{(1)}_{itv}\left(1-\theta^{(1)}_{itv}\right)\right\}^{1/2}$$

and

$$BU\left(\frac{1-\gamma}{2}\,;\, n_{ijt}\,,\, \theta^{(2)}_{itv}\right) \approx Z_{\frac{1+\gamma}{2}} \left\{n_{ijt}\theta^{(2)}_{itv}\left(1-\theta^{(2)}_{itv}\right)\right\}^{1/2} . \tag{3.8}$$

Program NAVYR is based on this approximation. It turned out, indeed, that program NAVYR determines the tolerance limits about five times faster than program NAVYS. The tolerance limits, however, do not differ in most of the cases in more than one unit. In Table 7 we provide the tolerance limits

Table 7

Tolerance Limits (γ=.95, 1-α=.95)
by Programs NAVYS and NAVYR: OSC 1

	NAVYS					NAVYR				
j/t	4	5	6	7	8	8	7	6	5	4
9					26 11	26 11				
10					14 3	13 4				
11				7 0	6 0	6 0	6 1			
12				89 57	83 50	83 50	89 58			
13			24 13	22 9	21 7	20 8	22 9	24 13		
14			9 2	8 1	8 1	8 1	8 2	9 3		
15		8 1	8 2	8 1	7 0	7 1	7 1	8 2	8 2	
16		49 29	53 34	47 26	44 22	43 23	46 26	53 35	48 30	
17	58 40	51 31	55 36	49 27	46 23	45 24	48 28	55 37	51 32	58 41
18	49 33	43 25	47 29	41 22	39 19	38 19	41 22	46 30	43 26	48 33

determined by programs NAVYS and NAVYR for the data of Table 1. As seen in Table 7, the approximation provided by program NAVYR is quite good and justifies the use of this program, which saves a considerable amount of computing time.

4. Numerical Forecasting of Pilot Retention

In the present section we provide the numerical results obtained by applying program NAVYR to the data of Tables 1-6. The results are given in Tables 8-11. Each table extends over ten values of j ($j = 9,\ldots,18$) which corresponds to cohorts entering from the first quarter of 1973. There are eight columns corresponding to values of t from 2 (one year before MSRI) to 8 (2 years after MSRI). For each value of j we present the upper tolerance limit (first row), lower tolerance limit (second row), and actual value (third row). In all cases where the actual values have not yet been observed we print the value o . These cases appear on the lower right side of each table. The other cases correspond to past periods. The tolerance limits were computed for past periods for the sake of testing whether the actual values fall in the tolerance intervals. As seen in the tables, the tolerance intervals indeed cover most of the actual values. Finally, the program adds the upper and lower tolerance limits of all cohorts for one, two, three, and four periods ahead to provide the prediction intervals for the number of pilots from those presently in service, from all the six OSC's, which will be in service 6, 12, 18, and 24 months from the forecasting date. For the present data under consideration these limits are

Table 12

Tolerance Limits For All Pilots From The 6 OSC's

	6m	12m	18m	24m
Upper	1519	1257	988	686
Lower	822	662	537	362

Table 8

Tolerance Limits For OSC 1 Cohorts
(γ=.95, 1-α=.95)

j/t	2	3	4	5	6	7	8
9	57	55	50	47	42	30	26
	53	50	35	34	32	16	11
	55	55	51	43	35	35	0
10	24	23	22	21	21	18	13
	22	20	13	13	14	8	4
	23	23	22	21	20	17	0
11	8	8	8	7	7	6	6
	7	6	3	3	4	1	0
	8	8	7	7	7	0	0
12	177	176	140	139	124	89	83
	169	164	112	111	102	58	50
	176	161	155	130	124	0	0
13	33	31	28	26	24	22	20
	30	27	18	16	13	9	8
	31	30	27	27	0	0	0
14	12	12	12	10	9	8	8
	10	10	6	5	3	2	1
	12	12	10	9	0	0	0
15	9	9	8	7	8	7	7
	8	7	3	2	2	1	1
	9	8	8	0	0	0	0
16	74	74	65	48	53	46	43
	70	67	47	30	35	26	23
	74	72	62	0	0	0	0
17	66	66	53	51	55	48	45
	62	60	41	32	37	28	24
	66	65	0	0	0	0	0
18	56	54	48	43	46	41	38
	52	49	33	26	30	22	19
	54	54	0	0	0	0	0

Table 9

Tolerance Limits For OSC 3 Cohorts
(γ=.95, 1-α=.95)

j/t	2	3	4	5	6	7	8
9	164	139	91	73	54	50	34
	141	122	64	55	36	34	17
	146	130	83	66	57	49	0
10	266	230	141	118	97	92	68
	235	207	106	93	71	68	41
	243	207	137	121	108	105	0
11	148	120	80	68	54	42	43
	126	104	55	50	36	23	23
	125	113	77	65	64	0	0
12	205	164	101	96	74	57	58
	179	145	72	74	53	34	34
	172	145	111	92	88	0	0
13	212	165	111	94	62	61	62
	185	146	80	73	39	36	36
	173	160	109	94	0	0	0
14	306	229	163	157	113	110	112
	272	206	124	127	78	74	74
	242	240	184	177	0	0	0
15	156	126	89	52	60	58	59
	133	110	63	31	37	35	35
	132	127	90	0	0	0	0
16	156	145	93	66	76	74	75
	133	127	66	41	49	46	46
	152	134	116	0	0	0	0
17	163	164	106	93	107	104	106
	140	145	74	62	73	69	69
	172	167	0	0	0	0	0
18	158	165	107	95	108	106	107
	136	146	76	63	75	71	71
	173	170	0	0	0	0	0

Table 10

Tolerance Limits For OSC 4 and OSC 6 Cohorts
(γ=.95, 1-α=.95)

j/t	2	3	4	5	6	7	8
9	41	40	37	29	30	21	15
	36	36	24	18	21	11	5
	40	40	32	31	23	18	0
10	47	47	41	33	30	21	15
	42	42	28	20	21	11	5
	47	45	36	31	23	18	0
11	93	92	78	66	55	40	39
	84	84	58	47	41	22	20
	92	88	76	57	52	0	0
12	37	37	32	29	24	18	17
	32	33	21	18	16	7	6
	37	35	32	24	21	0	0
13	12	12	12	11	9	9	9
	10	10	6	4	3	2	2
	12	12	11	10	0	0	0
14	24	24	23	17	13	13	13
	20	21	13	9	5	4	4
	24	24	18	15	0	0	0
15	64	59	51	32	35	34	33
	57	53	36	17	19	18	16
	59	57	43	0	0	0	0
16	60	55	47	34	37	36	35
	54	50	32	18	21	19	17
	55	52	46	0	0	0	0
17	32	31	26	22	23	23	22
	28	27	15	10	11	10	9
	31	28	0	0	0	0	0
18	34	32	29	25	26	26	25
	30	28	18	12	14	12	11
	32	32	0	0	0	0	0
9	18	15	12	8	7	7	7
	11	10	4	3	3	3	1
	15	13	8	7	7	7	0
10	33	29	24	18	14	13	12
	23	21	11	10	8	7	3
	29	27	18	14	13	13	0

(Cont'd)

Table 10

j/t	2	3	4	5	6	7	8
11	22 14 15	15 10 14	13 4 10	10 4 10	10 5 9	9 3 0	9 1 0
12	8 4 5	5 2 4	4 0 3	3 0 2	2 0 1	1 0 0	1 0 0
13	6 3 4	4 2 4	4 0 2	2 0 0	2 0 0	2 0 0	2 0 0
14	6 3 3	3 1 3	3 0 2	2 0 2	2 0 0	2 0 0	2 0 0
15	5 2 4	4 2 3	3 0 2	2 0 0	2 0 0	2 0 0	2 0 0
16	6 3 6	6 3 5	5 0 5	5 0 0	5 1 0	5 1 0	5 0 0
17	2 0 2	2 0 2	2 0 0	2 0 0	2 0 0	2 0 0	2 0 0
18	2 0 2	2 0 2	2 0 0	2 0 0	2 0 0	2 0 0	2 0 0

Table 11

Tolerance Limits For OSC 38 and OSC 80 Cohorts
(γ=.95, 1-α=.95)

j/t	2	3	4	5	6	7	8
9	20	18	15	12	11	10	9
	13	12	6	7	4	4	1
	18	16	12	11	10	10	0
10	17	16	13	7	7	7	7
	11	11	5	3	2	2	0
	16	13	7	7	7	7	0
11	13	11	10	9	8	8	7
	8	7	3	4	2	1	0
	11	10	9	8	8	0	0
12	12	10	9	6	6	6	6
	7	6	3	2	1	0	0
	10	9	6	6	6	0	0
13	13	10	10	8	7	7	7
	8	6	3	4	1	0	0
	10	10	8	7	0	0	0
14	21	18	17	13	12	12	11
	14	12	7	7	4	3	2
	18	18	13	13	0	0	0
15	8	7	6	6	6	6	6
	4	4	1	0	1	0	0
	7	6	6	0	0	0	0
16	7	6	4	3	3	3	3
	3	3	0	0	0	0	0
	6	4	3	0	0	0	0
17	2	1	1	1	1	1	1
	0	0	0	0	0	0	0
	1	1	0	0	0	0	0
18	1	1	1	1	1	1	1
	0	0	0	0	0	0	0
	1	1	0	0	0	0	0
9	22	20	15	13	10	9	5
	14	15	5	5	3	1	0
	20	18	13	10	10	8	0
10	21	19	16	16	13	11	8
	13	14	6	7	4	2	0
	19	19	16	14	13	13	0

Table 11

j/t	2	3	4	5	6	7	8
11	51	41	29	23	17	12	11
	37	32	13	12	6	1	0
	41	36	24	18	18	0	0
12	61	45	29	28	22	14	13
	45	35	13	16	9	2	1
	45	36	20	24	22	0	0
13	33	29	22	18	15	12	11
	22	22	9	9	4	1	0
	29	27	19	19	0	0	0
14	38	26	21	20	16	13	12
	26	19	9	10	5	2	0
	26	26	21	21	0	0	0
15	20	18	14	10	11	9	9
	12	13	5	2	2	0	0
	18	17	14	0	0	0	0
16	16	14	11	8	9	8	7
	9	10	3	1	1	0	0
	14	13	11	0	0	0	0
17	30	28	21	18	21	17	15
	20	21	8	6	7	3	1
	28	27	0	0	0	0	0
18	52	54	40	35	39	31	28
	38	43	20	15	17	9	6
	54	55	0	0	0	0	0

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APPENDIX

Computer Programs

1. Program NAVYS (FORTRAN)

```
      DIMENSION IX(6,13,3),P(6,7),PU(6,7),PL(6,7),IPU(6,7)
       DIMENSION IPL(6,7),ISL(10,4),FSU(4),FSL(4)
       CALL OPENF(1,"PIL1")
      CALL OPENF(2,"PIL3")
      CALL OPENF(3,"PIL4")
      CALL OPENF(4,"PIL5")
      CALL OPENF(5,"PI33")
      CALL OPENF(6,"PI30")
      Z=1.96
      GA=.975
      GB=.025
      N=6
      K=13
      M=3
      DO 1 J=1,K
      READ(1,10) (IX(1,J,L),L=1,M)
      READ(2,10) (IX(2,J,L),L=1,M)
      READ(3,10) (IX(3,J,L),L=1,M)
      READ(4,10) (IX(4,J,L),L=1,M)
      READ(5,10) (IX(5,J,L),L=1,M)
      READ(6,10) (IX(6,J,L),L=1,M)
   10 FORMAT(3I3)
    1 CONTINUE
      DO 175 J=1,4
      READ(50,176) (ISL(I,J),I=1,10)
  176 FORMAT(10I1)
  175 CONTINUE
      MM=M-1
      DO 177 I=1,4
      FSU(I)=0.
      FSL(I)=0.
  177 CONTINUE
      DO 100 I=1,N
      WRITE(66,110) I
  110 FORMAT(3X,I5)
      DO 6 J=9,K
      DO 7 L=1,MM
      LL=L+1
      GO TO 300
  200 U=0.
      V=0.
      DO 3 JS=1,K
      IF(IX(I,JS,LS)) 3,3,4
    4 U=U+IX(I,JS,LS)
      V=V+IX(I,JS,LL)
```

```
    3 CONTINUE
      P(I,L)=(U+.5)/(U+1.)
      R=SQRT((P(I,L)/(1.-P(I,L))))
      Y=2.*ATAN(R)
      SE=SQRT(1./U)
      CL=Y-Z*SE
      CU=Y+Z*SE
      PL(I,L)=((SIN(CL/2.))**2)
      PU(I,L)=((SIN(CU/2.))**2)
      IF(IFL) 350,350,360
  300 IF(IX(I,J,LL)) 8,8,9
    8 LS=L
   13 IF(IX(I,J,LS)) 11,11,12
   11 LS=LS-1
      GO TO 13
   12 TK=IX(I,J,LS)
      NU=TK
      LS=L-1
      IFL=0
      GO TO 200
  350 TU=PU(I,L)
      TL=PL(I,L)
      IPU(I,L)=KBU(GA,NU,TU)
      IPL(I,L)=KBL(GB,NU,TL)
      GO TO 7
    9 LS=L
      NU=IX(I,J,LS)
      IFL=1
      GO TO 200
  360 TU=PU(I,L)
      TL=PL(I,L)
      IPU(I,L)=KBU(GA,NU,TU)
      IPL(I,L)=KBL(GB,NU,TL)
    7 CONTINUE
      WRITE(66,30) J,(IPU(I,L),L=1,MM)
   30 FORMAT(5X,I3,7I5)
      WRITE(66,31) (IPL(I,L),L=1,MM)
   31 FORMAT(8X,7I5)
      WRITE(66,31) (IX(I,J,L),L=2,4)
      DO 600 IL=1,4
      LF=ISL(J-3,IL)
      IF(LF) 600,600,610
  610 FSU(IL)=FSU(IL)+IPU(I,LF)
      FSL(IL)=FSL(IL)+IPL(I,LF)
  600 CONTINUE
    6 CONTINUE
  100 CONTINUE
       WRITE(66,700)
  700 FORMAT(10X,13HTOTALS FORECASTING)
        WRITE(66,710) (FSU(I),I=1,4)
        WRITE(66,710) (FSL(I),I=1,4)
  710 FORMAT(5X,4F10.0)
```

```
      END
      FUNCTION KBL(P,N,T)
      G=P
      M=N
      W=T
      J=0
    1 FJ=CB(J,M,W)
      IF(FJ-G) 3,2,2
    3 J=J+1
      GO TO 1
    2 IF(J-1) 6,5,5
    5 J=J-1
    6 KBL=J
      RETURN
      END
      FUNCTION KBU(P,N,T)
      G=P
      M=N
      W=T
      J=0
    1 FJ=CB(J,M,W)
      IF(FJ-G) 2,3,3
    2 J=J+1
      GO TO 1
    3 KBU=J
      RETURN
      END
      FUNCTION CB(I,N,P)
      J=I
      M=N
      Q=P
      IF(M-50) 11,11,12
   11 AM=M
      R=1.-Q
      IF(J) 1,2,3
    1 CB=0.
      GO TO 10
    2 CB=R**M
      GO TO 10
    3 B=R**M
      CB=B
      DO 4 K=1,J
      AK=K
      B=B*Q*(AM-AK+1.)/(R*AK)
      CB=CB+B
    4 CONTINUE
      GO TO 10
   12 AM=M
      AJ=J
      Z=(AJ+.5-AM*Q)/SQRT(AM*Q*(1.-Q))
```

```
      C3=CNDX(Z)
   10 RETURN
      END
      FUNCTION CNDX(Y)
      X=Y
      ISWTCH=0
      IF(X) 1,2,2
    1 X=ABS(X)
      ISWTCH=1
    2 P=.2316419
      B1=.31938153
      B2=-.356563782
      B3=1.781477937
      B4=-1.821255978
      B5=1.330274429
      T=1./(1.+P*X)
      R=.3989423*EXP(-X*X/2.)
      QNDX=1.-R*(B1*T+B2*T*T+B3*(T**3)+B4*(T**4)+B5*(T**5))
      IF(ISWTCH) 3,4,3
    3 QNDX=1.-QNDX
    4 CNDX=QNDX
      RETURN
      END
```

2. Program NAVYR (FORTRAN)

```
      DIMENSION IK(6,13,3),P(6,7),PU(6,7),PL(6,7),IPU(6,7)
       DIMENSION IPL(6,7),ISL(10,4),FSU(4),FSL(4)
       CALL OPENF(1,"PIL1")
      CALL OPENF(2,"PIL3")
      CALL OPENF(3,"PIL4")
      CALL OPENF(4,"PIL6")
      CALL OPENF(5,"PI33")
      CALL OPENF(6,"PI36")
      Z=1.96
      N=6
      K=13
      M=3
      DO 1 J=1,K
      READ(1,10) (IK(1,J,L),L=1,M)
      READ(2,10) (IK(2,J,L),L=1,M)
      READ(3,10) (IK(3,J,L),L=1,M)
      READ(4,10) (IK(4,J,L),L=1,M)
      READ(5,10) (IK(5,J,L),L=1,M)
      READ(6,10) (IK(6,J,L),L=1,M)
   10 FORMAT(3I3)
    1 CONTINUE
      DO 175 J=1,4
      READ(50,176) (ISL(I,J),I=1,10)
  176 FORMAT(10I1)
  175 CONTINUE
      MM=M-1
      DO 177 I=1,4
      FSU(I)=0.
      FSL(I)=0.
  177 CONTINUE
      DO 100 I=1,N
      WRITE(66,110) I
  110 FORMAT(3X,I5)
      DO 6 J=9,K
      DO 7 L=1,MM
      LL=L+1
      GO TO 300
  200 U=0.
      V=0.
      DO 3 JS=1,K
      IF(IK(I,JS,LS)) 3,3,4
    4 U=U+IK(I,JS,LS)
      V=V+IK(I,JS,LL)
    3 CONTINUE
```

```
      P(I,L)=(V+.5)/(U+1.)
      R=SQRT((P(I,L)/(1.-P(I,L))))
      Y=2.*ATAN(R)
      SE=SQRT(1./U)
      CL=Y-Z*SE
      CU=Y+Z*SE
      PL(I,L)=((SIN(CL/2.))**2)
      PU(I,L)=((SIN(CU/2.))**2)
      IF(IFL) 350,350,360
  300 IF(IK(I,J,LL)) 3,3,9
    3 LS=L
   13 IF(IK(I,J,LS)) 11,11,12
   11 LS=LS-1
      GO TO 13
   12 TK=IK(I,J,LS)
      LS=L-1
      IFL=0
      GO TO 200
  350 UP=TK*PU(I,L)
      RUL=1.-PU(I,L)
      VP=UP*RUL
      SP=SQRT(VP)
      WP=UP+Z*SP
      IF(TK-WP) 21,22,22
   21 IPU(I,L)=INT(TK)
      GO TO 23
   22 IPU(I,L)=INT(WP)
   23 UP=TK*PL(I,L)
      RLL=1.-PL(I,L)
      VP=UP*RLL
      SP=SQRT(VP)
      WP=UP-Z*SP
      IF(TK-WP) 24,25,25
   24 IPL(I,L)=INT(TK)
      GO TO 7
   25 IPL(I,L)=INT(WP)
      GO TO 7
    9 LS=L
      TK=IK(I,J,LS)
      IFL=1
      GO TO 200
  360 RUL=PU(I,L)
      UP=TK*RUL
      VP=UP*(1.-RUL)
      SP=SQRT(VP)
      WP=UP+Z*SP
      IF(TK-WP) 51,51,52
   51 IPU(I,L)=INT(TK)
      GO TO 53
   52 IPU(I,L)=INT(WP)
```

```
   55 RLL=PL(I,L)
      UP=TK*RLL
      VP=UP*(1.-RLL)
      SP=SQRT(VP)
      WP=UP-Z*SP
      IF(TK-WP) 71,71,72
   71 IPL(I,L)=INT(TK)
       GO TO 7
   72 IPL(I,L)=INT(WP)
    7 CONTINUE
      WRITE(66,30) J,(IPU(I,L),L=1,MM)
   30 FORMAT(3X,I3,7I5)
      WRITE(66,31) (IPL(I,L),L=1,MM)
   31 FORMAT(3X,7I5)
      WRITE(66,31) (IK(I,J,L),L=2,M)
       DO 600 IL=1,4
      LF=ISL(J-3,IL)
      IF(LF) 600,600,610
  610 FSU(IL)=FSU(IL)+IPU(I,LF)
      FSL(IL)=FSL(IL)+IPL(I,LF)
  600 CONTINUE
    6 CONTINUE
  100 CONTINUE
      WRITE(66,700)
  700 FORMAT(10X,13HTOTALS FORECASTING)
       WRITE(66,710) (FSU(I),I=1,4)
       WRITE(66,710) (FSL(I),I=1,4)
  710 FORMAT(5X,4F10.0)
      END
```

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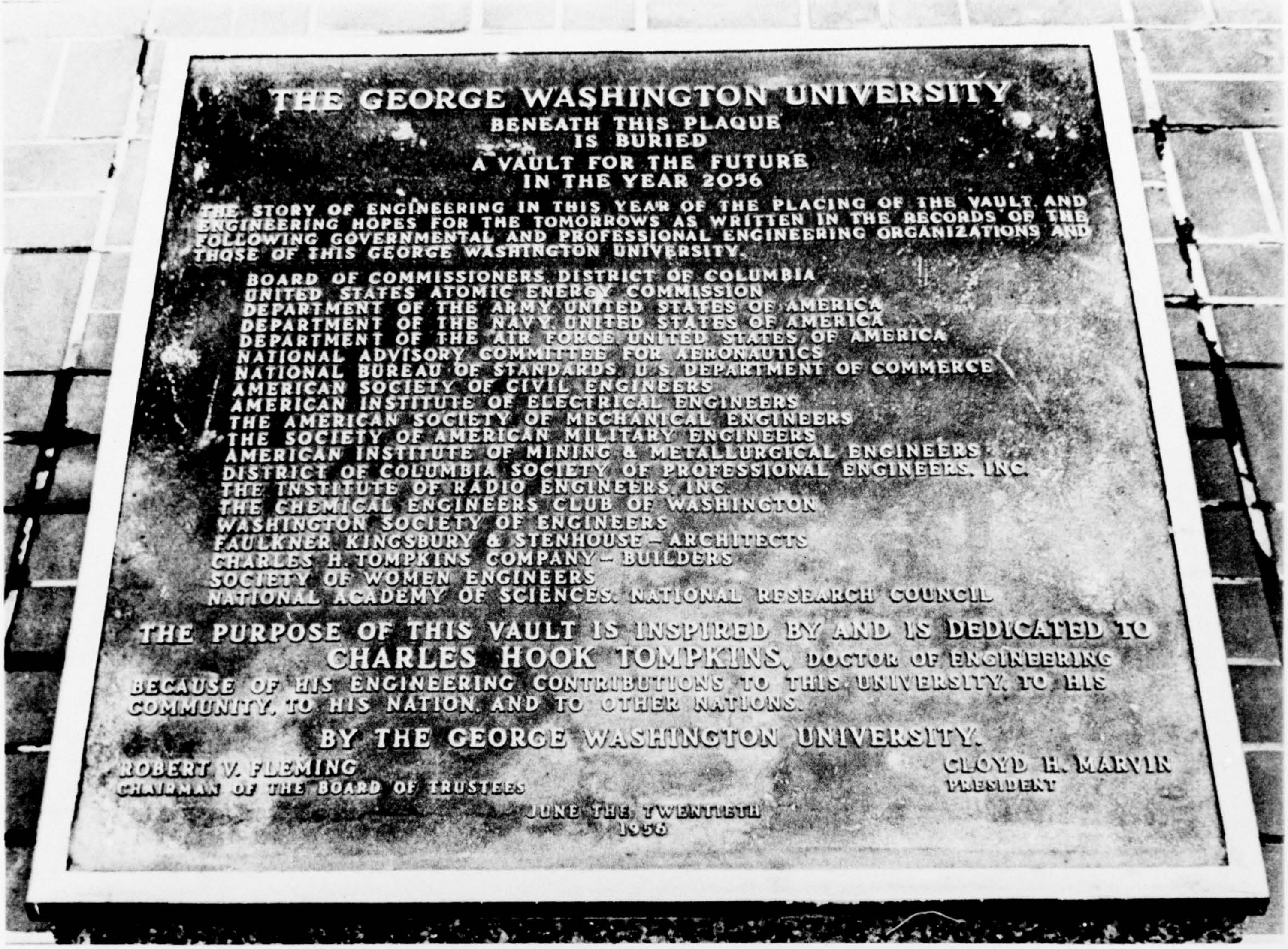

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